

APPLICATIONS OF LAPLACE SPHERICAL FUNCTIONS IN METEOROLOGY

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Key words: Laplace spherical functions, tidal potential, tidal cycles, cycles of the air temperature.

Abstract. The work tries to show that Laplace spherical functions can be applied in meteorology, too. In other words, some cycles in the evolution of maximum and minimum daily temperatures in the air can be explained by either the applications of these functions, which describe the tidal potential of the atmosphere. Tidal cycles of the atmosphere, explained by applying Laplace spherical functions, are reflected, through (planetary) Rossby waves, in cycles of maximum and minimum daily temperatures in the air, with the same duration. The 20 Laplace spherical functions describe, in a mathematical aspect, the tidal potential of the atmosphere, considering the Earth to be a sphere. Of these, the most important are the zonal, tesimal and sectoral spherical functions.

Introduction

Several cycles (almost perfect) in the evolution of maximum and minimum daily temperatures could be explained by atmospheric tidal cycles which, in their turn, are determined by the attraction of the Moon and Sun. These celestial bodies cause the tidal potential of the atmosphere, but also that of the lithosphere and ocean waters. The atmosphere and lithosphere are continue coatings, unlike ocean waters that form a discontinuous coating.

There are tidal cycles lasting less than one year and others lasting longer. All these cycles are based on elements in the evolution of the Moon and Sun. These elements work like independent variables in the distribution of the tidal potential of the atmosphere. By applying Laplace spherical functions that describe, under the mathematical aspect, the tidal potential of the atmosphere we obtain cycles in the evolution of the maximum and minimum daily temperatures with the same duration, as we shall demonstrate below.

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1. Laplace spherical functions applicable to the atmosphere

The 20 Laplace spherical functions, which describe, under the mathematical aspect, the tidal potential of the atmosphere are at the same time harmonic functions, too. Harmonic functions describe the oscillations occurring simultaneously in an environment without the possibility that some of them affect each other (without jam). This means that various forms of oscillations of the atmosphere generated by the attraction of the Moon and Sun and described by spherical Laplace harmonic functions do not influence each other. In the following, we shall describe only three Laplace spherical harmonic functions applicable to the atmosphere.

1.1. Zonal spherical function. This function mainly takes into account the latitude of a point on the Earth's surface subject simultaneously to the attraction of the Moon and of the Sun. The formula of this function is the following:

$y = 3 \sin^2 \varphi - 1$, in which φ is the latitude of the point of the Earth's surface. Nodal lines of this function (where the function gets zero) are the parallels of $+ 35^\circ 16'$ (North) and $- 35^\circ 16'$ (South). Here is the demonstration:

$$3 \sin^2 \varphi - 1 = 0, \text{ which means, } 3 \sin^2 \varphi = 1, \text{ from where}$$

$$\sin \varphi = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1,732}{3} = 0,577$$

The angle whose sinus = 0,577 is of $35^\circ 16'$, which means the parallels of $35^\circ 16'$ N and S.

Figure 1 presents the nodal lines of this function. The flow occurs between the two parallels $35^\circ 16'$ (N and S), and the low tide around the two poles to the nodal lines.

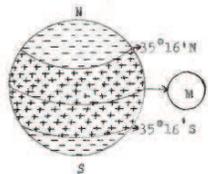


Fig. 1 - The repartition of the tidal potential of the zonal function
M = the disturbing celestial body (the Moon or the Sun)

From the constant part of this function, that is the permanent tide, an equipotential surface results, fallen by 28 cm at the two poles and raised by 14 cm at the Equator, if we relate to the oceanic waters. In the case of the atmosphere, the falling and raising of the equipotential surface have much higher values.

The duration of this tidal type is of approximately 14 days for the Moon and about 6 months for the Sun. This phenomenon is explained by the fact that no

matter what the declination sign of the Moon and the Sun (+ for the north hemisphere; – for the south hemisphere), at the same absolute value, the tides are identical in both hemispheres northern and southern).

The duration of approximately 14 days, in the Moon's case, is actually of 13,66 days, meaning half of the duration of the tropical revolution of the Earth's natural satellite, equal with 27,32 days ($27,32 : 2 = 13,66$). This highlights the daily value of the Moon's declination.

Fig. 2. represents the daily values of the Moon's declination, during a complete tropical revolution.

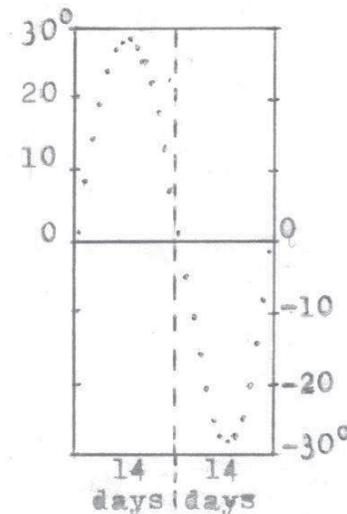


Fig. 2 - The daily values of the Moon's declination

Therefore, the time span in which Moon's declination rises from 0° to $+28^\circ 36'$ (North) and lowers back to 0° is identical with the period in which the Moon's declination lowers from 0° to $-28^\circ 36'$ (South) and rises to 0° . Each of these two periods lasts for approximately 14 days (13,66 days).

Identical periods can begin and end with other values of the Moon's declination.

In many of these situations, the cycle of 14 days also reflects in the maximum and minimum daily temperatures. This fact is true for any point or surface on the Earth, no matter what the latitude is.

Fig. 3. presents cycles of 14 days (almost perfect) in the evolution of the daily maximum and minimum temperatures recorded at several weather stations in Romania and Chatham Island (New Zealand).

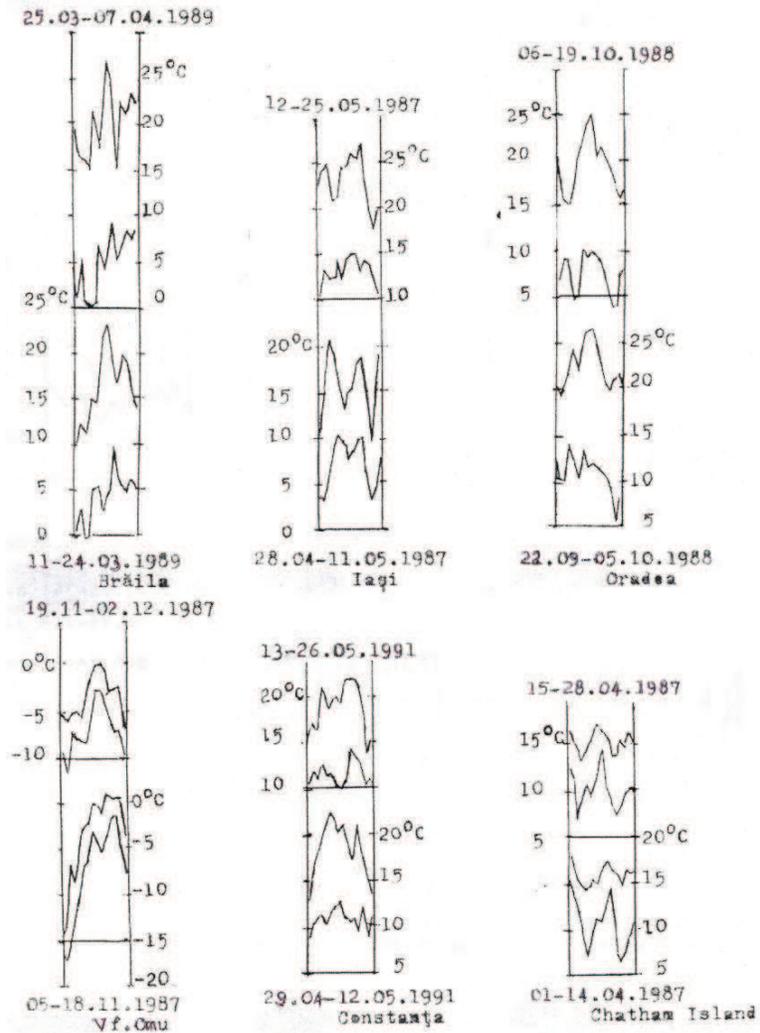


Fig. 3 - Cycles of daily maximum and minimum temperatures for 14 days recorded at weather stations in Romania and Chatham Island

In most situations, the 14-day cycles of thermal nature are accompanied by cycles of atmospheric pressure (daily averages) with the same length.

Figure 4 shows the 14-day cycle of atmospheric pressure (daily mean values at the meteorological station) between the 11th and the 24th of March 1989, compared with the 25th of March and the 7th of April 1989 period, the same cycle as in Figure 3 (for Braila), but for the maximum and minimum daily temperatures.

This is a demonstration of those previously stated.

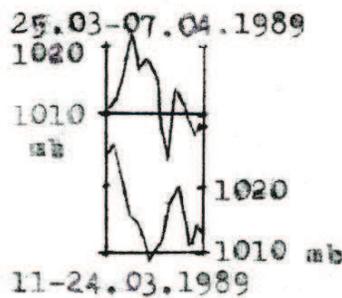


Fig. 4 - A fourteen days cycle of the atmospherical pressure in Braila

The cycles with the duration of 14 days, both those of the pressure of the atmosphere and those of the temperature of the air, are not perfect, because the tidal potential of the atmosphere is influenced by other independent variables too, of which the distance from the Moon to the Earth is more important. The anomalistic period (revolution), the one which indicates this distance, has a duration of 27,55 days. This means the time passed between the two successive passages of Selene through Perigee (the minimum distance to the Earth) or through Apogee (the maximum distance to the Earth). This period can't be halved as the tropical revolution (of the period) of the Moon (27,32 days), which imprints the cycles of 14 days. Under the distance aspect, the period from Perigee to Apogee is not identical to the period from Apogee to Perigee.

Here is how the distance Moon - Earth varied in the period 11.13 - 07.04. 1989 (cycles in fig. 3, Braila and fig.4). On 11.03.1989, the distance was of 363.900 km (parallax = 60'15''), then it grew up to the Apogee, on 22.03.1989, at 406.400 km (parallax = 53'58''). From 22nd March 1989 the distance decreased again up to 357.200 km (parallax = 61'23''), at Perigee on 05.04.1989, reaching on 07.04.1989, 358.600 km (parallax = 61'09'').

Next, we shall analyze the cycle of about 6 months (182,62 days), specific to the Sun, taking into consideration the daily values of its declination. Like the Moon, the Sun is considered the perturbation celestial body which generates a tidal potential of the atmosphere, but with a period of about 6

months (183 days). This tidal potential is reflected, through the planetary waves (Rossby), in the daily evolution of the maximum and minimal temperatures, with the same length.

Fig. 5 presents the daily values of the Sun declination during a tropical year (365,24 days). The period of 6 months can begin and end having the value of 0° , but also having any another value too, for instance, of $23^\circ 27'$, at solstice.

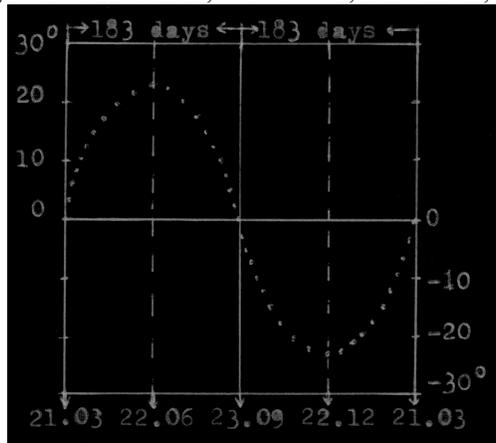


Fig. 5 - The values of the Sun declination during a tropical year

The 14 day cycles (the Moon case) and that of 183 days (the Sun case) are, after all, harmonic oscillations, because they do not influence each other.

As a consequence of these factors, cycles of daily maximum and minimum temperatures appear, accompanied by cycles of atmospheric pressure (daily averages) with the same length (about 6 calendar months).

Cycles with duration of 6 calendar months of daily maximum and minimum temperatures can occur anywhere on Earth, regardless of latitude. Figure 6 shows such cycles of daily maximum and minimum temperatures during the 6 calendar months.

As for the Moon (at the 14 day cycle), cycles of 6 calendar months, as demonstrated in the evolution of daily maximum and minimum temperatures in the graphs in Figure 6, are accompanied by cycles of atmospheric pressure (daily averages) with the same duration. Thermal cycles and the nature of the atmospheric pressure are not perfect, because there are also other variables involved in the evolution of the Sun (in fact, the Earth), including the Earth-Sun distance. Earth-Sun distance Aphelion, in early July is higher than at Perihelion, in early January.

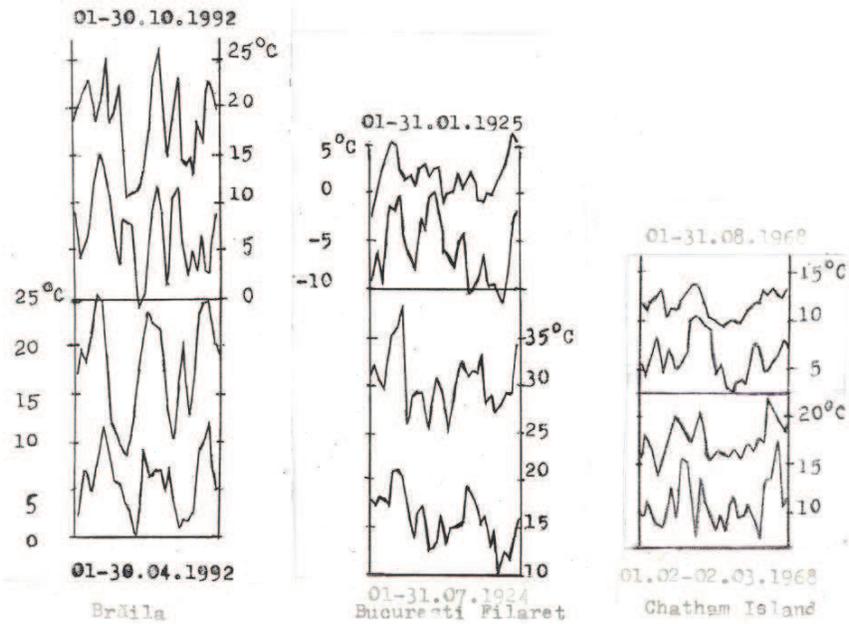


Fig. 6 - The 6 month cycles of the daily maximum and minimum temperatures in Brăila, Bucharest Filaret and Chatham Island

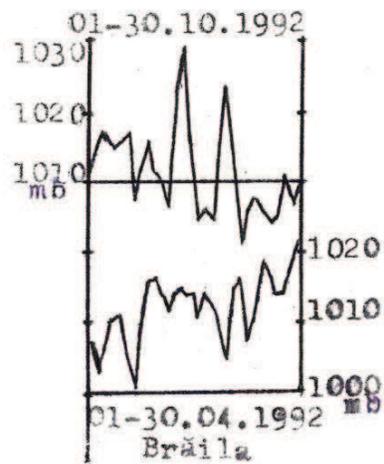


Fig. 7 - The cycle of six calendar months for atmospheric pressure

Figure 7 shows the cycle of 6 calendar months, when atmospheric pressure (daily averages) for the period 1 to 3 April 1992 is compared with the period 1 to 3 October 1992. Obviously, it is about the same period presented in the graphs in Figure 6 (in Braila), where we demonstrated the nature of the thermal cycle.

In accordance with the mathematical theory related to analysis and synthesis of harmonic functions, you can find cycles which take longer for both air temperature and atmospheric pressure. These cycles should be, by their duration, multiple cycles of 14 (13,66 days) and 183 days (182,62 days). In other words, these cycles must contain an integer number of half-year (182,62 days); an integer number of half of the Moon's tropical period (13,66 days); an integer number of half of the Moon's synodical period (14,76 days). The Moon's synodical period is of 29,53 days. Also, these cycles must be multiples of the period of the Moon anomalistic (27,55 days), because it cannot be halved. Obviously, there are cycles containing an even number of half a year, that is full years, and cycles with an odd number of half-year. From the first category, the closest to perfection is the cycle of four years, about 1461 days. Calculations show that:

- 1461: 13,66 = 106,995; about 107 halves of the Moon's tropical period.
- 1461: 14,76 = 98,984; about 99 halves of the Moon's synodical period.
- 1461: 27,55 = 53,031; about 53 anomalistic Moon periods.
- 1461: 182,62 = 8,000; 8 halves of years (4 years).

In some cases, cycles of 8, 11, 19, 23 and 27 years are confirmed. The 11-year cycle coincides with the solar activity cycle and the one of 19 years is known as the metonian cycle.

In the category of cycles that contain an odd number of half-year, the closest to perfection is the duration of the one of 13,5 years, about 4931 days. Calculations show that:

- 4931: 13,66 = 360,981; about 361 halves of the Moon's tropical period.
- 4931: 14,76 = 334,078; about 334 halves of the Moon's synodical period.
- 4931: 27,55 = 178,984; about 179 anomalistic Moon periods.
- 4931: 182,62 = 27,001 half years (13,5 years)

In some cases, we can confirm the cycles of 5,5 years, 9,5 years, 17, 5 and 21,5 years.

Of those listed above, and considering that the spherical functions Laplace are harmonics functions too, it means that the cycle of 14 days and 6 months can occur after a period of four years, but even after a 13,5 year period.

Because of the changes suffered by the local and regional *physic-geographical* conditions, cycles of 14 days and 6 months are not perfect. This is because meteorological measurements are made at a height of 2 m above the soil

surface, that is the active surface, not the one suffering changes from one year to another. However, the two cycles can be pointed out pretty well.

In fig. 8. is presented a cycle of 14 days (the same of figure 3 for Braïla), which is found in the cycle of four years and further in the cycle of 13,5 years. The cycles are evident in air temperature and atmospheric pressure (daily averages).

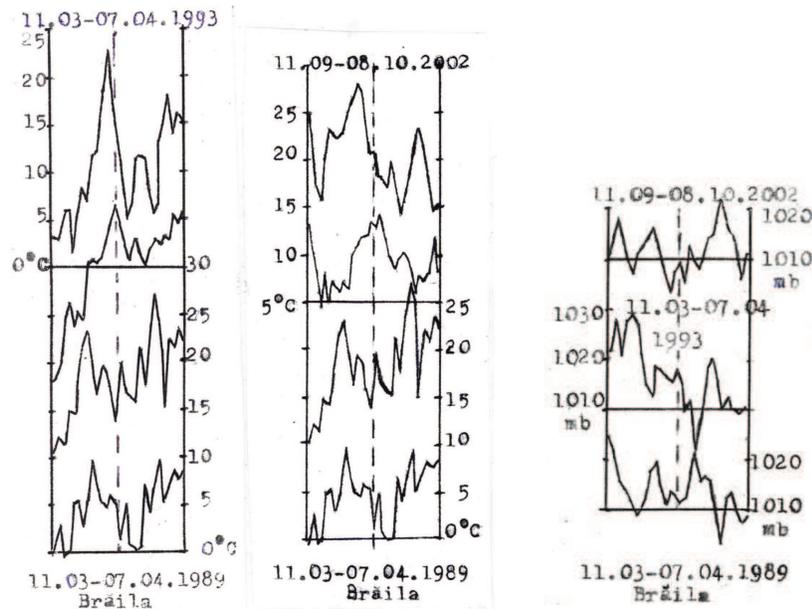


Fig. 8 - Cycles of 14 days, 4 years and 13,5 years for the maximum and minimum daily temperatures and atmospheric pressure (daily averages) at the meteorological station Braïla

As you can see, in these cycles there are some small differences in time (phase relationship). This is explained by the fact that although the Moon anomalistic period (27,55 days) is included in an integer number of times (it is a divisor) in the cycles of 4 years, and respectively, of 13,5 years (see previous calculations), lunar parallax values (which tells us the Earth-Moon distance) are different. The Perigee, at every anomalistic period, has another value of the parallax. As you know, the Perigee runs a revolution sidereal time of 3232,6 days. This is how the anomalies in astronomy from the Moon anomalistic period are reflected in meteorology.

Similarly, a cycle of 6 months (attraction generated by the Sun) can be found after four years, but also after a period of 13,5 years.

In figure nine, a six-month cycle is presented that can be found in the four-year cycle and finally, in the 13,5 – year cycle. The cycles are evident at both thermic and baric oscillations.

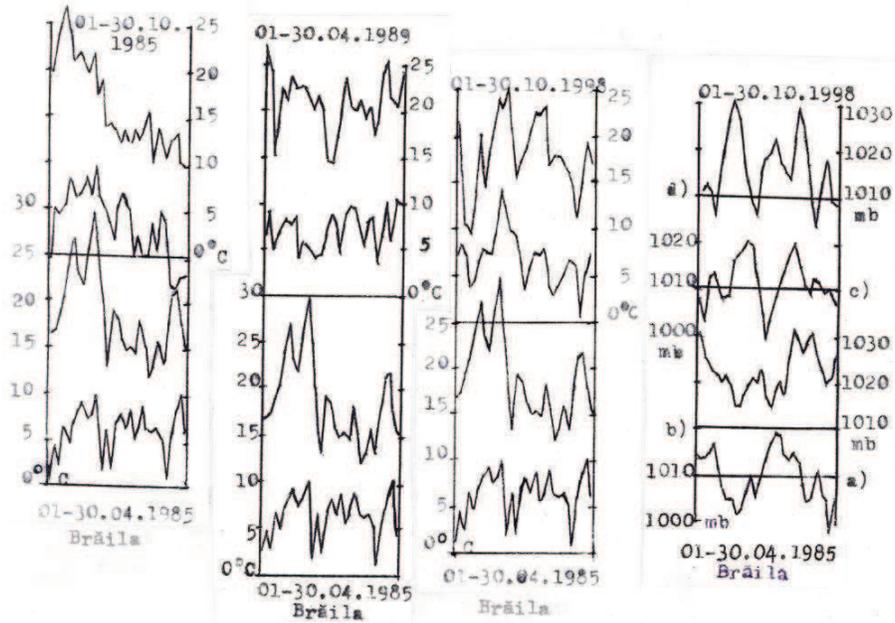


Fig. 9 - The cycles of six calendar months, the cycles of four and 13,5 years for the maximum and minimum daily temperatures and for the atmospheric pressure (daily averages), at Braila Meteorological Station.

Pressure's "a" graphs - the period between the 1st and the 30th of April 1985;

"b" is the period between the 1st and the 30th of October 1985;

"c"- the period between the 1st and the 30th of April 1989;

"d" is the period between the 1st and the 30th of October 1998.

1.2. The tesseral spherical function. This function takes into account the latitude of one point situated on the Earth's Surface and on the Moon's hour angle. The function's form is the next $y = \sin 2\varphi \cos H$, where " φ " is the point's latitude of the Earth's Surface and " H " is the Moon's hour angle. The nodal lines of this function are the Equator ($\varphi = 0^\circ$), because $\sin (2 \times 0^\circ) = \sin 0^\circ = 0$, but also the meridians of 90° situated on both sides of the meridian where the Moon is found at a certain time. If the Moon is located at the 90° meridian (as the drawing in Fig. 10. A), then the nodal lines are the Greenwich meridian and the 180° meridian. The

nodal lines are those lines where the function gets zero value, so at $\cos 90^\circ = 0$. This way, the Earth's Surface is divided into four equal parts, in which the tidal potential is alternatively positive and negative (+ and -), as in fig. 10 B.

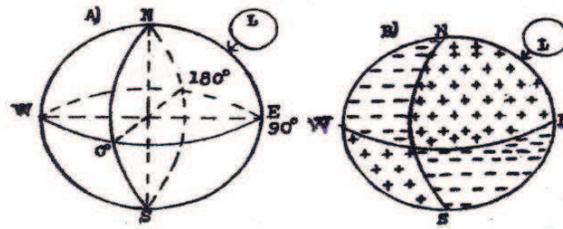


Fig.10 - The nodal lines (A) and the tidal potential's distribution (B) of the tesseral function $y = \sin 2\varphi \cos H$; L = The Moon

These portions are symmetrical to the equator and meridians from 0° to 180° . Tidal amplitude in this case occurs at the latitude of 45° (N and S) because $\sin(2 \times 45^\circ) = \sin 90^\circ = 1$, maximum value. It means that two points located at 45° North and South respectively, have identical tidal potential, if the two points have a longitudinal difference between them of 180° . We have the same result and a great potential in terms of shape similarity planetary waves (Rossby). In turn, they cause similar baric fields in the two points. Baric similar fields, determine further similar atmospheric circulation that ultimately reflected the evolution in terms of maximum and minimum daily temperatures of the two points.

On Earth's surface, we can choose two points to meet the conditions listed above. For example, the meteorological station Chatham Island (New Zealand), with geographic coordinates $43^\circ 57' S$, $176^\circ 34' V$, and the meteorological station Bordeaux Merignac (France), with the geographic coordinates $44^\circ 50' N$ and $0^\circ 42' W$. The requirements of the tesseral function are not perfectly met as far as the geographic position of the two points is concerned. For comparison, we took into consideration meteorological stations in Romania and Chatham Island, New Zealand (the French refused to provide meteorological data).

Fig 11. presents the daily maximum and minimum temperatures of the Chatham Island, for the period 01.06 – 31.08.1968, in comparison with their evolution in Oradea.

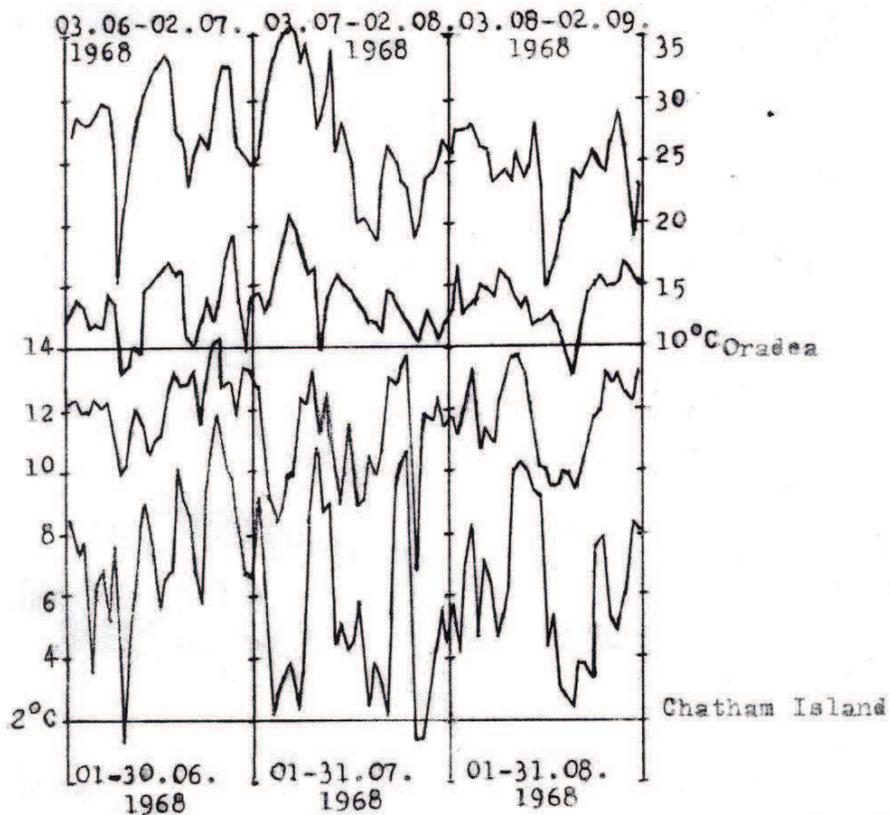


Fig 11 - Comparison between the daily maximum and minimum temperatures of the Chatham Island (the period 01.06 - 31.08.1968), and those of Oradea in the period 03.06 - 02.09.1968

The similarities are more than obvious, but the difference in time is of 2 days. This is explained by the time it takes the planetary waves, which are identical at Chatham Island with those of Bordeaux (about 13 degrees longitude per day) to propagate from the western France to the west of Romania. The difference in time of two days would disappear if instead of the Oradea weather station would be the Bordeaux Merignac weather station.

For the villages located in eastern Romania (but around 45° N latitude), this difference increased to 3 days. The graphs in Figure 12 show the evolution of maximum and minimum daily temperatures of Chatham Island in June, July,

November and December 1987 compared with the evolution of the Braila station of the June 4 to August 3 and 4 November 1987 - 03 January 1988 periods.

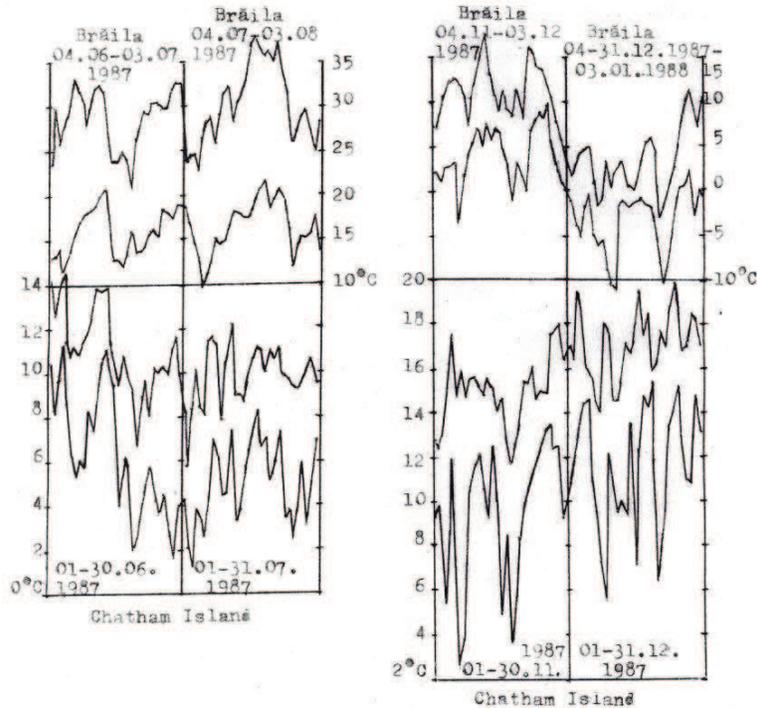


Fig. 12 - Comparison between the evolution of maximum and minimum daily temperatures of Chatham Island in June, July, November and December 1987 and the evolution of those in Braila in the period 04/06 to 03/08/1987 and, respectively, 04/11/1987 to 03/01/1988

As we can see, the evolution of maximum and minimum daily temperatures at the two meteorological stations shows many similarities, but also a difference in time of 3 days, a period needed for the propagation of the planetary waves (Rossby) from the West of France to the East of Romania.

1.3 Sectoral spherical function. This function has the form $y = \cos^2 \varphi \cos 2H$, where φ and H have the same meanings as those in the tesseral function, i.e. φ = declination point on the Earth's surface and H = hour angle of the Moon. Nodal lines of this function are the meridians of 45° on both sides of the meridian on which the Moon is at that time. If the Moon is at longitude 45° E, as in Fig. 13 a), the nodal lines are the meridians of 0° (Greenwich) and 90° E, and

their extensions, i.e. the meridian of 180° and 90° V. The demonstration is as follows:

To Moon hour angle (H) $\cos(2 \times 45^\circ) = \cos 90^\circ = 0$.

For the declination point on the Earth's surface $\cos^2 \varphi = 0$.

But $\cos^2 \varphi = \frac{1}{2}(1 + \cos \varphi) = \frac{1}{2} + \frac{\cos \varphi}{2}$, but $\frac{1}{2} + \frac{\cos \varphi}{2} = 0$, when $\cos \varphi = -1$, i.e. $\varphi = 180^\circ$ ($2 \times 90^\circ$), because $\cos 180^\circ = -1$, which means the two poles.

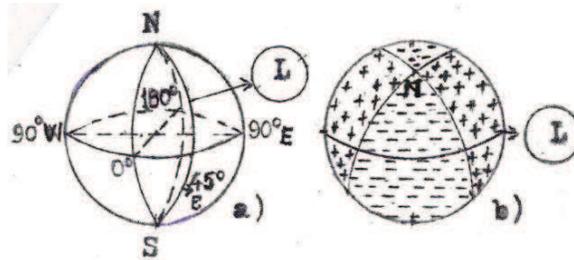


Fig. 13 - The nodal lines (a) and the tidal potential's distribution of the sectoral function $y = \cos^2 \varphi \cos 2H$; L = The Moon (b)

The function gets maximum value at the Equator (0°), because $\cos 0^\circ = 1$, maximum value, but at the declination of the Moon around 0° , i.e. when Selena was in the level of the Equator. Consequently, at the Equator, the tides have maximum amplitude.

Of the previously demonstrated, it results that two points located on the Equator, but at a longitudinal difference of 90° , have the tidal potential of atmosphere identical. This shows that in the two points, planetary waves have very similar shape. Furthermore, baric fields and identical circulations of the atmosphere result, and, finally, the evolution of maximum and minimum daily temperatures in the two points will be very similar.

It means that in cities like Iquitos, Kinshasa and Jakarta, located near the Equator and at a longitudinal distance of about 90° , the evolution of maximum and minimum daily temperatures will be very similar, at least at certain times of the year.

Conclusions

The Laplace *spherical harmonic* functions describe, mathematically speaking, the tidal potential of the atmosphere, and that could be applied in meteorology;

The atmospheric tides features are transmitted to the baric field of the atmosphere by (Rossby) planetary waves;

The similar baric field, resulted from the cyclical periods of the atmospheric tides, determines the circulation of the atmosphere, which, in its turn, is reflected in the daily progress of maximum and minimum temperatures by almost perfect cycles;

The cycles of the daily maximum and minimum temperatures of 14 days, 6 months and 13,5 years are a demonstration of the cyclic way in which these phenomena and processes take place in the atmosphere. The Moon is the main celestial body responsible for this evolution;

Knowing these cycles could help us develop long term weather forecasts (over 10 days);

Knowing the atmospheric lunar, solar and lunar-solar cycles helps us discover weather cycles (almost perfect) with the same duration and demonstrates the connection, within certain limits, between astronomy and meteorology.

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